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Estimating countermeasure effects for reducing collisions at highway–railway grade crossings

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Abstract

Frequently transportation engineers are required to make difficult safety investment decisions in the face of uncertainty concerning the cost-effectiveness of different countermeasures. For certain types of highway–railway grade crossings, this problem is further aggravated due to the lack of observed before and after collision data that reflects the impact of specific countermeasures. This study proposes a Bayesian data fusion method as an attempt to overcome these challenges. In this framework, we make use of previous research findings on the effectiveness of a given countermeasure, which could vary by jurisdictions and operating conditions to obtain a priori inference on its expected effects. We then use locally calibrated models, which are valid for a specific jurisdiction, to develop the current best estimates regarding the countermeasure effects. By using a Bayesian framework, these two sources are integrated to obtain the posterior distribution of the countermeasure effectiveness. As a result, the outputs provide information not only of the expected collision response to a specific countermeasure but also its variance and corresponding probability distribution for a range of likely values. Examples from Canadian highway–railway grade crossing data are used to illustrate the proposed methodology and the specific effects of prior knowledge and data likelihood on the combined estimates of countermeasure effects.

Keywords: Highway–railway grade crossing; Safety; Countermeasure; Bayesian data fusion

1. Introduction

Laughland et al. (1975) introduced the concept of collision modification factor (CMF) to reflect the safety benefits associated with different countermeasures and to represent the expected changes in collisions resulting from their introduction. The CMF can be expressed simply as the ratio of the expected (or observed) number of collisions after the countermeasure is introduced at a given site to the expected number before its introduction.

The FHWA developed a series of CMF for two-lane rural highways (Harwood et al., 2000; Zegeer et al., 1992). The forthcoming US Highway Safety Manual will provide a series of CMF to reflect the effect of different design and operational strategies applied to highways (Hughes et al., 2004; Harkey, 2005). In the highway–railway grade crossing field, the term CMF has not been used extensively. Many researchers have preferred to use the expected reduction in collisions resulting from a given safety intervention (Farr, 1987; FRA, 2002; Saccomanno and Lai, 2005). While these two terms are generally equivalent, we will use CMF to be consistent with the convention of road safety research.

The estimation of CMF requires a sound and accurate estimate of the expected number of collisions for a specific crossing "before and after" the introduction of a given countermeasure. Over the past two decades, a number of approaches have been employed for predicting collisions and estimating the effectiveness of countermeasures; the most popular being cross-sectional and before-and-after models.

Saccomanno and Lai (2005) suggested that there are a number of unresolved statistical issues inherent in conventional single-stage cross-sectional models, including variable co-linearity, misspecification of inputs, failure to consider higher-order interactions, treatment selection biases, and regression-to-the-mean, etc.

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In before–after models, two types of approaches have been adopted: naïve and Empirical Bayesian (EB) models. The EB before-and-after models were introduced to resolve many of the "regression-to-the-mean (RTM)" biases associated with the naïve approach. An in-depth discussion on this issue is provided by Abbess et al. (1981), Hauer and Persaud (1987), and Wright et al. (1988) and Gan et al. (2005).

While EB before and after models are expected to reduce much of the RTM bias inherent in the conventional naïve approach, Lord (2006) and Park and Saccomanno (in press) argue that these types of models fail to reflect other biases introduced by the rarity of the event being considered. Grade crossing collision data is normally plagued by "too many zero reported collisions", and this places special restrictions on the use of the EB method for predicting collisions at specific grade crossings based on a limited time interval over which observations are obtained.

As discussed by Melcher et al. (2001), even though numerous data and methodological issues yield inconsistencies of results regarding the effect of different countermeasures, previous model results may still be useful. Conducting new studies for every single countermeasure of specific interest in resolving specific local problems is impractical. The challenge posed by Melcher is "not to throw these estimates out, but rather to systematically and objectively integrate relevant findings from different sources to provide a comprehensive appreciation of countermeasure effects applied to different transportation safety problems and locations".

In summary, regardless of the model adopted a significant degree of bias in the estimates of countermeasure effects can be introduced. This is due to a number of reasons, including:

- Rarity and randomness of collisions.
- Lack of adequate statistical controls or misspecification of factors.
- Complex co-linearity issues among variables.
- Treatment selection biases (e.g. the RTM problem).
- Data aggregation.
- Data reporting biases.

In light of these biases, in this paper we have taken the view that countermeasure effects can best be treated as random variables and their distributions should be obtained formally by integrating prior distributions with location-specific data.

This paper has four basic objectives:

(1) Identify and provide estimates of countermeasure effects as reported in the literature from different sources or studies. This serves to provide a "prior" belief on the nature of the effects as obtained from these studies.
(2) Introduce a formal data fusion method for integrating estimates of countermeasure effects from previous studies (1) with those from in-depth data analysis for the region of interest (in this case grade crossing collision data from the Canadian railway network).
(3) Integrate formal treatment of uncertainty in the estimates of countermeasure effects.
(4) Investigate the effectiveness of selected countermeasures as applied to a given crossing with a mix of relevant attributes. In this paper, we illustrate the proposed approach using two types of countermeasures: elimination of whistle prohibition and upgrading of warning devices.

2. Bayesian data fusion

This study proposes a Bayesian data fusion method for combining countermeasure effects from different independent sources with estimates obtained from a formal analysis of the grade crossing data. The proposed approach is similar to that suggested by El Faouzi (2006), Melcher et al. (2001) and Washington and Oh (2006), but different in how prior knowledge and data likelihood functions are developed.

In this paper, our aim is to obtain "posterior" estimates of the probability of the effect induced by a given countermeasure applied to a specific crossing i with a given mix of attributes. The posterior expression is of the form (Migon and Gamerman, 1999; Lee, 2004):

$$P_i(\theta|x) \propto P_i(\theta)P_i(x|\theta) \tag{1}$$

where θ is the countermeasure effect (CMF) for a specific crossing; x the estimate from Canadian collision prediction models; $P_i(\theta)$ the prior probabilities of θ from past studies; $P_i(x|\theta)$ the probability of observing the sample data given that a statement about the value of a parameter is true (i.e. objective or current best knowledge); $P_i(\theta|x)$ the posterior probability of θ give x.

Eq. (1) assumes that the effect of a given countermeasure is best treated as a random variable with a unique probability distribution. Since these estimates are obtained from independent sources and are commonly empirical in nature, we assume that for a given crossing they are normally distributed with a given mean and a variance. As noted by Lee (2004), the observations which have a built-in estimation error are likely to reflect a normal distribution according to the central limit theorem.

If the distribution of multiple source estimates on the priors and data likelihoods are normal the posterior estimates are also normal. Note that this normal distribution assumption is purely for computational convenience and other distributions are equally applicable with the proposed data fusion method. The use of other distributions may require more computationally intensive procedures such as, Markov Chains Monte Carlo (MCMC) techniques. Similar to Washington and Oh (2006), more flexible beta distribution, which can explain the nonsymmetric nature of countermeasure effects, is also considered in this study in obtaining the posterior probability distribution for a given countermeasure effect.

2.1. Prior and data likelihood distributions

As noted above from Melcher et al. (2001) estimates of countermeasure effects based on previous studies represent a "first order a priori" belief concerning their values in the absence of a formal data analysis. Since each source is assumed to yield a separate "independent" estimate of the effect, these estimates can be represented by a unique "a priori" probability distribution. In this paper, we assume historical knowledge from a number of previous studies regarding similar countermeasures based on different jurisdictions. Many of these sources are based on research involving US data. In this paper, we have assumed that the Canadian and US experience are close enough to justify the assertion that countermeasure effects come from the same statistical population.

While prior estimates are assumed to be independent, their accuracy is subject to the reliability and strength of the method adopted for predicting collisions. Obviously, some methods improve on the shortcomings of other methods, and these would need to be given higher weights when obtaining a "combined" a priori effect.

In this research, we follow a similar approach to that adopted by Harkey (2005) and Washington and Oh (2006) to establish the relative weights of countermeasure effects based on the perceived merits of different model types. In general, we obtained the relative study weight (i.e. W_{ij} in Eq. (2)) as the inverse ranking of the level of certainty summarized in Table 1 for different types of analysis methods.

The mean combined countermeasure effect from previous studies is obtained using a weighted average expression of the form:

$$\mu_j = \frac{\sum W_{ij}\mathrm{CMF}_{ij}}{\sum W_{ij}} \tag{2}$$

where CMF_{ij} is the effectiveness of countermeasure j in level of certainty i; W_{ij} the relative study weight for countermeasure j in level of certainty i; μ_j is the weighted average effectiveness of countermeasure j from all available sources.

To obtain the prior distributions for countermeasure effects, we need to obtain the variance as well as the mean associated with this effect. Estimates of the mean are routinely provided in the various sources. Unfortunately, estimates of CMF variance tend to be unavailable since many sources fail to provide empirical estimates of variance for different countermeasure effects.

In the absence of specific information on countermeasure CMF variance for a given prior source, we have suggested the following five step procedure:

(1) Obtain the mean countermeasure effect (μ_j) as well as the standard deviation (σ_j) for countermeasure j from all sources that provide these two pieces of information.
(2) Estimate the "coefficient of variation" for countermeasure j using an expression of the form:

$$\mathrm{CV}_j = \frac{\sigma_j}{\mu_j}100 \tag{3}$$

where σ_j is the standard deviation of the countermeasure j; μ_j the CMF of the countermeasure j; CV_j is the coefficient of variation for the countermeasure j.
(3) Obtain "average CV" for the countermeasures in the same level of certainty i (as per Table 1).
(4) Apply "average CV" obtained from the method being used regardless of type of countermeasure and estimate its associated standard deviation for the countermeasure by using Eq. (3).
(5) Assign relative study weights in Table 1 to the individual countermeasure and combine its estimated standard deviation to obtain weighted average standard deviation for a specific countermeasure j based on the Eq. (4).

$$\sigma_j = \frac{\sum W_{ij}\sigma_{ij}}{\sum W_{ij}} \tag{4}$$

where σ_{ij} is the standard deviation of countermeasure j in level of certainty i; W_i the relative study weight for countermeasure j in level of certainty i; σ_j is the weighted average standard deviation of countermeasure j from all available sources.

For the purpose of illustration, a numerical example of approximating standard deviation for a selected countermeasure (i.e. upgrading from signboards to flashing lights) is provided as follows. For this countermeasure, three previous studies, that is, Morrissey (1980), Eck and Halkias (1985) and Farr and Hitz (1985), have provided estimates of the mean and standard deviance, as given in Table 2. The following steps are taken to estimate the mean and standard of CMF for the particular crossing of our analysis:

(1) By applying Eq. (3), for a given CV calculation the following example calculations were carried out for three different sources. From Morrissey (1980), CV was estimated as 11.43 (=0.04/0.35 × 100). The CV was estimated to be 5.16 and 7.97 from Eck and Halkias (1985) and Farr and Hitz (1985), respectively. The average CV from these sources was calculated as 8.19 [=(11.43 + 5.16 + 7.97)/3].
(2) By applying this average CV to the other six studies available for the same level of certainty, we can approximate standard deviation (τ) for these studies. For instance, the Alaska State DOT reported a mean CMF value of 0.25

Table 1
Relative weight based on study method

Level of certainty (i)	Brief description of study methodology	Relative study weight (W_i)
1. High	Empirical Bayesian (EB) before–after models with proper application	1.00
2. Medium–High	Sound before–after (but not EB before–after) or cross-sectional models with rigorous expert judgment. Combination of study results using rigorous Meta-analysis	0.50
3. Medium–Low	Cross-sectional models with controlling for other factors statistically or naïve before–after models	0.33
4. Low	Before–after or cross-sectional models in which modeling technique were questionable	0.25

Table 2
Estimated Priors for Improvement from Signboards to Flashing Lights

Level of certainty	μ	τ	CV	Literature
Medium–Low (0.33)[a]	0.35	0.0400	11.43	Morrissey (1980)
	0.31	0.0160	5.16	Eck and Halkias (1985)
	0.29	0.0231	7.97	Farr and Hitz (1985)
	0.25	0.0205	8.19	Alaska State[b]
	0.62	0.0507	8.19	Arizona State[b]
	0.23	0.0188	8.19	Idaho State[b]
	0.50	0.0409	8.19	Iowa State[b]
	0.35	0.0286	8.19	Kentucky State[c]
	0.35	0.0286	8.19	Missouri State[b]

[a] Relative Study Weight.
[b] Gan et al. (2005).
[c] Agent et al. (1996).

without reporting the standard deviation. Using the Eq. (3), we estimated this standard deviation to be 0.0205 ($\approx 8.19 \times 0.25/100$).

Based on a thorough review of previous grade crossing studies, we obtained a weighted average of the CMF and variance for 18 different countermeasures. Table 3 summarizes these estimates along with the number of studies or sources on which they are based. These will be used to represent the historical information or a priori belief as to the effectiveness of countermeasures in the absence of any analyses involving the actual collision data.

Based on the results of these previous studies, the strongest countermeasure effects (excluding grade separation or closure) is an upgrade in warning device from 2- to 4-Quadrant Gates and the installation of Photo/Video enforcement. Both countermeasures are believed to reduce grade crossing collisions by about 75%. On the other hand, the weakest effect was found for the introduction of yield signs ahead of grade crossings. The expected collision reduction for this countermeasure was estimated to be about 19%. For these results, a total of 91 sources were investigated to obtain a priori countermeasure effects (Saccomanno et al., 2006).

It should be noted here that the formal "Meta Analysis" approach proposed in literature to integrate findings from multiple studies also utilizes the same expression as Eqs. (2) and (4) to estimate the weighted average and variance of existing findings (Hunter and Schmidt, 1990). But a major difference concerns the estimation of relative study weights from previous studies. Furthermore, the "Meta Analysis" method requires a number of inputs from each study, including sample size, published year, omitted factors, and even the number of researchers. Recently, White (2002) attempted to represent the effectiveness of 30 different safety countermeasures on the basis of Meta analysis; however, only 5 different countermeasures effects were obtained due to the lack of necessary input information. Unfortunately, the input information required for a rigorous Meta analysis of grade crossing countermeasure effects was not available from the previous studies cited in Table 3. Our aim in this analysis is to produce estimates of effectiveness for as many countermeasures as possible; hence a formal Meta analysis was not

Table 3
Estimated CMF and variance from the past studies

Number	Countermeasures	μ	τ	No. of previous studies
1	Grade Separation/Closure	0.0000	0.0000	2
2	Yield Sign	0.8100	0.0723	4
3	Stop Sign	0.6467	0.0577	6
4	Stop Ahead Sign	0.6533	0.0583	3
5	Stop Line Sign	0.7200	0.0642	3
6	Illumination (Lighting)	0.5625	0.0502	4
7	Pavement Markings	0.7914	0.0706	7
8	From Signs to Flashing Lights	0.4578	0.1356	10
9	From Signs to 2Q-Gates	0.2833	0.0864	10
10	From Flashing Lights to 2Q-Gates	0.4738	0.1489	7
11	From 2Q-Gates to 2Q-Gates with Median Separation	0.3375	0.0301	4
12	From 2Q-Gates to 4Q-Gates	0.2540	0.0227	5
13	Installing Traffic Signal	0.3583	0.1776	4
14	Elimination of Whistle Prohibition	0.4671	0.0417	3
15	Improve Sight Distance	0.6630	0.0591	10
16	Improve Pavement Condition	0.5200	0.0464	3
17	Posted Speed Limit	0.8000	0.0714	3
18	Photo/Video Enforcement	0.2471	0.0220	3
				Sum = 91

employed in this study. The proposed approach remains practicable in that the priors can be easily updated or altered should better results become available from future studies concerning specific countermeasures.

Prior estimates may not be reliable because they are study specific and limited in reflecting the full gamut of crossing-specific factors that we would expect to influence collisions at different locations in different jurisdictions. For this analysis, we require an in-depth investigation of the relationship between crossing attributes and collisions as reflected in the Canadian database. The estimated CMF from these collision prediction models best represents the "objective" or current information for grade crossing collisions as well as attributes within Canadian jurisdictions. From the Bayesian perspective, we refer to this type of inference as "data likelihood".

In this study, we employed three different statistical models based on independent studies carried out by Saccomanno and Lai (2005) and Park and Saccomanno (2005a,b). These models were developed for Canadian grade crossing data adopting a multi-stage cross-sectional approach to reduce many of problems associated with conventional cross-sectional models.

Saccomanno and Lai (2005) introduced a three-stage cross-sectional model to predict collisions at grade crossings. They grouped crossings into five different clusters with similar attributes based on sequential factor/cluster analyses and then developed cluster-specific collision prediction models using negative binomial expressions. Since crossing attributes within individual clusters are assumed to be homogenous, the expected change in the number of collisions before and after the introduction of a given countermeasure can be used to assess its effect.

Park and Saccomanno (2005a) introduced a data partitioning method (i.e. RPART) to eliminate the impact of different control factors, which can influence collisions but are difficult for engineers to alter directly (e.g. jurisdictional attributes). The model focuses on the impact of the countermeasures themselves. The authors assigned individual crossings into homogenous groups of crossings in terms of selected control factors, and then developed a series of statistical models to predict collisions and corresponding countermeasure effects.

Park and Saccomanno (2005b) attempted to introduce higher-order interaction terms in their prediction model, employing a data partitioning method to reflect higher-order interactions, which could not be captured using conventional cross-sectional models.

The three multi-stage data likelihood models based on the aforementioned three previous studies to produce data likelihoods are summarized in Tables 4–6 for each model, respectively. Note that these collision prediction models all employ negative binomial expressions (except Class 4 model in Table 5), and the exposure term in the models is expressed as the product of the "number of daily trains" and "road volume (AADT)".

2.2. Posterior distribution

If the estimate from one of three data likelihood models is given as x_1 with probability $P_1(x_1|\theta)$, then we can estimate the posterior distribution for this estimate from Eq. (1).

Table 4
Cluster-specific negative binomial collision prediction models based on Saccomanno and Lai model (2005)

Variables	Coding scheme	Cluster 1		Cluster 2		Cluster 3		Cluster 4		Cluster 5	
		Coefficient	S.E.	Coefficient	S.E.	Coefficient	S.E.	Coefficient	S.E.	Coefficient	S.E.
Flashing lights (FL)	FL = 1; Otherwise = 0	−0.383	0.326	−0.994	0.188	–	–	–	–	−0.580	0.212
Gates (GT)	GT = 1; Otherwise = 0	−0.840	0.293	−1.448	0.344	–	–	–	–	−1.492	0.324
Active warning signs	FL or GT = 1; sign = 0	–	–	–	–	−1.119	0.137	−1.222	0.208	–	–
Maximum train speed	Mile/h	–	–	0.016	0.004	0.011	0.003	–	–	–	–
Track angle	Degree	–	–	–	–	−0.009	0.006	–	–	–	–
Whistle prohibition (WP)	WP = 1; Otherwise = 0	–	–	1.152	0.334	–	–	–	–	0.807	0.164
Ln(Exposure)	Ln (AADT × Daily Train)	0.358	0.040	0.387	0.043	0.461	0.032	0.441	0.037	0.497	0.059
Intercept		−4.057	0.373	−4.967	0.290	−4.537	0.481	−4.309	0.306	−6.071	0.525
Dispersion (ϕ)		0.794	0.190	1.112	0.312	0.361	0.213	0.973	0.216	1.614	0.328

Table 5
Class-specific collision prediction models based on Park and Saccomanno model (2005a)

Variables	Coding scheme	Class 1		Class2		Class3		Class4		Overall Class	
		Coefficient	S.E.	Coefficient	S.E.	Coefficient	S.E.	Coefficient	S.E.	Coefficient	S.E.
Flashing lights (FL)	FL = 1; Otherwise = 0	−0.677	0.147	−0.571	0.235	−0.983	0.131	–	–	−0.756	0.084
Gates (GT)	GT = 1; Otherwise = 0	−0.899	0.185	−0.601	0.205	−1250	0.236	–	–	−1.004	0.114
Surface type	If Paved = 1; Unpaved = 0	–	–	−0.254	0.155	-0.22	0.124	–	–	−0.112	0.067
Whistle prohibition (WP)	IfWP = 1; Otherwise = 0	0.294	0.114	–	–	0.827	0.174	1.409	0.780	0.373	0.085
Maximum train speed	km/h	0.002	0.001	–	–	0.007	0.002	0.011	0.005	0.004	0.001
Ln(Exposure)	Ln(AADT × Daily Train)	0.345	0.030	0.358	0.048	0.366	0.033	0.290	0.077	0.355	0.020
CI01		n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	−3.867	0.173
CI02		n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	−4.004	0.172
CI03		n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	−3.965	0.150
CI04		n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	−4.388	0.169
Intercept		−3.797	0.266	−3.821	0.368	−4.190	0.241	−4.789	0.421	n.a.	n.a.
Dispersion (ϕ)		0.633	0.114	0.236	0.180	0.439	0.154	n.a.	n.a.	0.543	0.082

Note: Class 1 represents the crossings at arterial or collector roads (NB model). Class 2 represents the crossings at local or other road types with multiple tracks (NB model). Class 3 represents the crossings at local roads with single track (NB model). Class 4 represents the crossings at other road types with single track (Poisson model).

Table 6
Negative binomial collision prediction models with group indicators based on Park and Saccomanno model (2005b)

Variables	Coding scheme	Coefficient	S.E.
Flashing lights (FL)	FL = 1; Otherwise = 0	−0.728	0.096
Gates (GT)	GT = 1; Otherwise = 0	−0.912	0.118
Maximum train speed (MTS)	Medium level MTS: 36 < MTS < 92 km/h = 1; otherwise = 0	0.274	0.086
	High level MTS: MTS > 92 km/h = 1; otherwise = 0	0.316	0.092
Ln(Exposure)	Ln(AADT × Daily Train)	0.422	0.019
GI08	C11 takes value 1 if a crossing installed with active warning devices (flashing lights or gates), in arterial or collector or local roads, with paved surface, with multiple track; Otherwise = 0	0.144	0.087
GI11	C11 takes value 1 if a crossing installed with signs, with medium level train speed, with non-perpendicular track angle; Otherwise = 0	0.409	0.127
GI13	C13 takes value 1 if a crossing installed with signs, with medium level train speed, with non-perpendicular track angle, with posted speed under 85 km/h; Otherwise = 0	−0.234	0.140
Intercept		−4.609	0.170
Dispersion (ϕ)		0.554	0.083

For a different experiment with estimate of x_2 with probability $P_2(x_2|\theta)$, we obtain the posterior probability as $P(\theta|x_2, x_1) \propto P(\theta)P_2(x_2|\theta)P_1(x_1|\theta)$. Generalizing this procedure for n different experiments, Migon and Gamerman (1999) derived the expression for the posterior probability as:

$$P(\theta|x_n, x_{n-1}, \ldots, x_1) \propto P(\theta)\left[\prod_{i=1}^{n} P_i(x_i|\theta)\right] \tag{5}$$

The technical challenge here is to obtain posterior probability distributions by integrating multiple distributions as per Eq. (5). From Bayes' theorem, if we assume normality in both the prior $[\theta \sim N(\mu, \tau^2)]$ and data likelihood distributions $[l \sim N(x, \sigma^2)]$, Lee (2004) and Migon and Gamerman (1999) demonstrated that it is possible to combine their means and variances analytically to produce a normal posterior distribution for $\theta|x$, with a mean of μ_0 and a variance of τ_1^2, where:

$$\mu_0 = \omega\mu + (1 - \omega)x \tag{6}$$

in which,

$$\omega = \frac{\tau^{-2}}{\tau^{-2} + \sigma^{-2}} \in (0, 1) \tag{7}$$

$$\tau_1^2 = (\tau^{-2} + \sigma^{-2})^{-1} \tag{8}$$

$$\mu_0 = (\tau^{-2}\mu + \sigma^{-2}x)\tau_1^2 \tag{9}$$

The ω in Eq. (7) measures the relative information contained in the prior with respect to its posterior distribution. As a result, Eq. (6) yields the combined weighted means of prior and data likelihood.

3. Example applications

To illustrate the application of the proposed data fusion method, we consider the following two types of countermeasures:

1. Introducing whistles at a crossing where whistles are currently prohibited.
2. Upgrading warning devices from Flashing Lights to 2-Quadrant Gates.

Table 7 summarizes the crossing attributes used in this numerical example. Data likelihood countermeasure effects are estimated using the three prediction models introduced above. In this example, we will report only on the results of the cluster/factor analysis model. This model requires that we first estimate factor scores based on crossing attributes, and then use these scores to assign crossings to individual clusters (groupings of attributes). To shorten the illustration, we will not describe how cluster membership is determined for a specific crossing, but simply indicate which cluster is involved. In this analysis, the given crossing belongs to Cluster 5 in both before and after countermeasure states.

By applying the Cluster 5 collision prediction expression (from Table 4), we obtain the CMF estimate before and after

Table 7
Sample Crossing Attributes for Example Applications

Variables	Crossing attributes	
	Before	After
Road surface width (ft)	15	15
Surface material	Asphalt (Paved)	Asphalt (Paved)
Road type	Local	Local
Track number	Single	Single
Track angle (degree)	70	70
Mainline or non-mainline	Mainline	Mainline
Average annual daily traffic (AADT)	15,000	15,000
Number of daily trains	12	12
Posted speed limit (km/h)	50	50
Maximum train speed (mile/h)	10	10
Whistle[a]	Prohibited	Operated
Warning devices[b]	Flashing lights	Flashing lights

[a] Countermeasure for the first example application.
[b] Countermeasure for the second example application.

the elimination of "whistle prohibition" as follows:

$$\mathrm{CMF_{WO}} = \frac{E\{N_{\mathrm{ai}}\}}{E\{N_{\mathrm{bi}}\}} = \frac{\exp(0.807.0)}{\exp(0.807.1)} \approx 0.446$$

Based on the Saccomanno and Lai's model, for Cluster 5 crossings we can expect a 55.4% reduction in collisions after whistle operations are introduced to this crossing.

By applying the delta method (Sampson, 2006; Xu and Long, 2005), the variance of $\mathrm{CMF_{WO}}$ can be approximated by $\mathrm{Var}(\mathrm{CMF_{WO}}) = (\mathrm{CMF_{WO}})^2\mathrm{Var}(\hat{\beta}_{\mathrm{ai}} - \hat{\beta}_{\mathrm{bi}})$. In this particular example, the estimated variance is simply equal to the square of the estimated standard error of the coefficient corresponding to the elimination of whistle prohibition (Table 4). This becomes $(0.164)^2 \approx 0.027$. The approximated variance of $\mathrm{CMF_{WO}}$ becomes $(0.446)^2\, 0.027 \approx 0.005$ (i.e. standard errors $\approx$0.073). As a result, the estimated $\mathrm{CMF_{WO}}$ follows $N(0.446, 0.073^2)$.

By applying the same procedure to expression for Cluster 3 in Table 5 we obtained second model estimates (i.e. $N(0.437, 0.076^2)$) for inputs into the data likelihood. However, we decided not use the third model in Table 6, since this model failed to explain variation in collision for this specific countermeasure (elimination of whistle prohibition). Inasmuch as we still have two point estimates, we can estimate the data likelihood distribution. In fact, this situation illustrates one of the merits in the proposed method. If we can estimate at least one CMF and its corresponding variance, we can still produce the data likelihood associated with a specific crossing and generate its posterior distribution.

Since we obtained $P_1(x_1|\theta) = N(0.446, 0.073^2)$, and $P_2(x_2|\theta) = N(0.437, 0.076^2)$, the data likelihood can be estimated using Eqs. (8) and (9), such that:

$$\tau_1^2 = (0.073^{-2} + 0.076^{-2})^{-1} \approx 0.053^2$$

$$\mu_0 = (0.076^{-2}0.437 + 0.073^{-2}0.446)0.053^2 \approx 0.442$$

As a result, the estimated data likelihood distribution for this specific crossing follows $N(0.442, 0.053^2)$, and this would represent the objective or current best knowledge about the expected effectiveness of the elimination of whistle prohibition for this specific crossing. On the other hand, the subjective or historical a priori belief for the same countermeasure was given in Table 3 as $N(0.467, 0.042^2)$.

Consequently, given the prior (i.e. $N(0.467, 0.042^2)$) and the data likelihood (i.e. $N(0.442, 0.053^2)$) distributions, we can produce the posterior distribution by applying Eqs. (5), (8), and (9), such that:

$$\tau_1^2 = (0.042^{-2} + 0.053^{-2})^{-1} \approx 0.033^2$$

$$\mu_0 = (0.042^{-2}0.467 + 0.053^{-2}0.442)0.033^2 \approx 0.457$$

If we wish to represent the contribution of the prior to the posterior, we apply Eq. (7) to yield the relative information parameter ω, such that:

$$\omega = \frac{0.042^{-2}}{0.042^{-2} + 0.053^{-2}} \approx 0.616$$

As a result, the expected reduction in collisions at this grade crossing due to the elimination of whistle prohibition was estimated to be approximately 55%. The contribution of prior information (ω) to the posterior distribution [$N(0.457, 0.033^2)$] was estimated to be in the order of 62%.

The second countermeasure example discussed in this paper deals with the introduction of 2-Quadrant Gates to a given crossing currently equipped with Flashing Lights. All other factors are assumed constant. For this exercise we assumed that the whistle prohibition is in effect for both types of warning devices.

Three different point estimates were obtained for the data likelihood based on the three models: (1) $N(0.402, 0.095^2)$, (2) $N(0.765, 0.161^2)$, and (3) $N(0.833, 0.069^2)$. In Table 3, prior distribution for this countermeasure was reported to be $N(0.474, 0.149^2)$. By applying a series of Eqs. (5)–(9), we obtained final posterior result for this countermeasure, namely $N(0.669, 0.050^2)$, for a value of ω equal to 0.111. This countermeasure resulted in a 33.1% reduction in the expected number of collisions with about 11.1% of this reduction being explained by the prior distribution alone.

These results are interesting in that, for this specific crossing, the elimination of whistle prohibition, which is usually treated as a supplementary countermeasure, produces higher safety benefits than were obtained for the upgrade in warning device from Flashing Lights to Gates. Had we tried to infer the safety benefits based solely on a priori belief, the effectiveness of these two countermeasures would be similar, both reflecting a 53% reduction in collisions.

This result does not necessarily mean that the elimination of whistle prohibition yields higher safety dividends than a more costly installation of gates at crossings with flashing lights for all crossings involved. We must view this result as being applicable only to this specific crossing of interest. The approach outlined in this paper is useful in that it provides a tailored CMF for specific crossing attributes.

4. Impact of different type of distribution in Bayesian data fusion

In this study, we have employed normal density functions to represent both of prior and posterior distributions. However as was noted previously, the normal distribution is symmetrical and unbounded while the effectiveness of a countermeasure may follow a skewed distribution with values bounded within a certain range (e.g. between 0 and 1). In this section, we investigated the impact of our normality assumption by assuming a beta distribution for both prior and data likelihood. Previous researchers (e.g. Clarke and Sarasua, 2003; Washington and Oh, 2006) have suggested using a beta distribution to represent both priors and posteriors because of its flexibility in representing a wide range of distribution patterns. Law and Kelton (1991) also indicated that the beta distribution is very useful in exploring a given dataset in the absence of any specific information about the dataset. One property of the beta distribution is that it must be bounded over a given interval of likely values. In this study, the beta distribution was defined over the range [0,1] for collision reductions of 100 and 0%, respectively.

Since the purpose of our study is to produce a varying CMF based on grade crossing attributes rather than estimate the average effectiveness of each countermeasure, an analytic method was used to combine different beta prior and beta data likelihood estimates.

The beta distribution was defined in terms of two shape parameters α and β, both greater than zero (Iversen, 1984), such that:

$$Pr(X = x; \alpha, \beta) = \frac{(\alpha + \beta - 1)!}{(\alpha - 1)!(\beta - 1)!} X^{\alpha-1}(1 - X)^{\beta-1} \tag{10}$$

where,

$$\mu = \frac{\alpha}{\alpha + \beta} \quad \text{(i.e. mean of beta distribution)} \tag{11}$$

$$\sigma^2 = \frac{\mu(1-\mu)}{\alpha + \beta + 1} = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)} \quad \text{(i.e. variance of beta distribution)} \tag{12}$$

By solving for α and β:

$$\alpha = \mu \left[\frac{\mu(1-\mu)}{\sigma^2} - 1 \right] \tag{13}$$

$$\beta = [1 - \mu] \left[\frac{\mu(1-\mu)}{\sigma^2} - 1 \right] \tag{14}$$

As pointed out by Harlow et al. (1997), basic advantage of the beta distribution is that the posterior beta parameters are additive functions of the beta prior and beta likelihood parameters, such that:

$$\alpha_{\text{posterior}} = \alpha_{\text{prior}} + \alpha_{\text{data likelihood}} \tag{15}$$

$$\beta_{\text{posterior}} = \beta_{\text{prior}} + \beta_{\text{data likelihood}} \tag{16}$$

Table 8
Sample mean and variance of prior and data likelihood

	Prior	Factor/cluster model (D1)	Stratified model (D2)
Mean (μ)	0.467	0.446	0.437
Variance (σ^2)	0.042^2	0.073^2	0.076^2

After obtaining the posterior beta parameters (i.e. $\alpha_{\text{posterior}}$, $\beta_{\text{posterior}}$), the mean and the variance of the posterior distribution can be estimated using Eqs. (11) and (12).

In the previous numerical example for the elimination of whistle prohibition at a given crossing, estimates of the mean and variance of prior and data likelihood were obtained as summarized in Table 8.

First, Eq. (5) and Eqs. (10)–(16) were applied to the data to yield the beta data likelihood estimates of CMF, such that:

(1) For the Saccomanno and Lai model (we call this D1):

$$\alpha_{\text{D1}} = \mu_{\text{D1}}\left[\frac{\mu_{\text{D1}}(1-\mu_{\text{D1}})}{\sigma^2_{\text{D1}}} - 1\right] = 0.446\left[\frac{0.446(1-0.446)}{0.073^2} - 1\right] \approx 20.240$$

$$\beta_{\text{D1}} = [1-\mu_{\text{D1}}]\left[\frac{\mu_{\text{D1}}(1-\mu_{\text{D1}})}{\sigma^2_{\text{D1}}} - 1\right] = [1-0.446]\left[\frac{0.446(1-0.446)}{0.073^2} - 1\right] \approx 25.140$$

(2) For the Park and Saccomanno (a) model (we call this D2):

$$\alpha_{\text{D2}} = \mu_{\text{D2}}\left[\frac{\mu_{\text{D2}}(1-\mu_{\text{D2}})}{\sigma^2_{\text{D2}}} - 1\right] = 0.437\left[\frac{0.437(1-0.437)}{0.076^2} - 1\right] \approx 18.041$$

$$\beta_{\text{D2}} = [1-\mu_{\text{D2}}]\left[\frac{\mu_{\text{D2}}(1-\mu_{\text{D2}})}{\sigma^2_{\text{D2}}} - 1\right] = [1-0.437]\left[\frac{0.437(1-0.437)}{0.076^2} - 1\right] \approx 23.210$$

(3) The $\alpha_{\text{datalikelihood}}$ and $\beta_{\text{datalikelihood}}$ from Eqs. (15) and (16) are:

$$\alpha_{\text{data likelihood}} = 20.240 + 18.041 = 38.281$$

$$\beta_{\text{data likelihood}} = 25.140 + 23.210 = 48.350$$

(4) To estimate the expected mean (x) and variance (σ^2) of the data likelihood distribution we use Eqs. (11) and (12), such that:

$$x_{\text{data likelihood}} = \frac{38.281}{38.281 + 48.350} \approx 0.442$$

$$\sigma^2_{\text{data likelihood}} = \frac{0.442(1-0.442)}{38.281 + 48.350 + 1} \approx 0.053^2$$

Since the beta distribution is assumed for the estimated prior (i.e. $B(0.467, 0.042^2)$) and the estimated data likelihood (i.e. $B(0.442, 0.053^2)$), the beta posterior distribution was obtained using the following steps:

(1) Estimate α_{prior} and β_{prior}, as such:

$$\alpha_{\text{prior}} = \mu_{\text{prior}}\left[\frac{\mu_{\text{prior}}(1-\mu_{\text{prior}})}{\sigma^2_{\text{prior}}} - 1\right] = 0.467\left[\frac{0.467(1-0.467)}{0.042^2} - 1\right] \approx 66.499$$

$$\beta_{\text{prior}} = [1-\mu_{\text{prior}}]\left[\frac{\mu_{\text{prior}}(1-\mu_{\text{prior}})}{\sigma^2_{\text{prior}}} - 1\right] = [1-0.467]\left[\frac{0.467(1-0.467)}{0.042^2} - 1\right] \approx 75.853$$

(2) $\alpha_{\text{data likelihood}}$ and $\beta_{\text{data likelihood}}$ is already estimated 38.281 and 48.350, respectively.

(3) Obtain $\alpha_{\text{posterior}}$ and $\beta_{\text{posterior}}$, as such:

$$\alpha_{\text{posterior}} = 66.499 + 38.281 = 104.780$$

$$\beta_{\text{posterior}} = 75.853 + 48.350 = 124.200$$

(4) Estimate the expected mean and variance of CMF, as such:

$$\mu_{\text{posterior}} = \frac{104.780}{104.780 + 124.200} \approx 0.458$$

$$\sigma^2_{\text{posterior}} = \frac{0.458(1-0.458)}{104.780 + 124.200 + 1} \approx 0.033^2$$

As a result, the estimated CMF follows $B(0.458, 0.033^2)$.

Fig. 1 and Table 9 provide the results of a comparison between the normal and beta cumulative posterior distributions and their corresponding parameters, respectively. Several observations can be made:

(1) The two cumulative distributions are almost identical and produce the same percentile values in the wide range of CMF values. The 5, 25, 50, 75, and 95 percentile values of the two distributions are estimated about 0.404, 0.435, 0.457, 0.480, and 0.512, respectively. As a result, there is a 5% of chance that the estimated CMF from the elimination of whistle prohibition is under 0.404, suggesting more than a 60% reduction in collisions. Similarly, there is a 5% chance

Table 9
Comparison between normal and beta posterior distribution

Distribution	Statistics for each countermeasure	Elimination of whistle prohibition	Upgrading flashing lights to gates
Normal	Mean (μ)	0.4574 (54.26)[a]	0.6693 (33.07)[a]
Beta		0.4576 (54.24)[a]	0.6181 (38.19)[a]
Normal	Standard errors (σ)	0.0327	0.0496
Beta		0.0329	0.0576

[a] Represents the percentage value of the estimated collision reduction.

that we can obtain less than a 49% of reduction for the same countermeasure.

(2) Contrary to the elimination of whistle prohibition, notable discrepancy is observed in the cumulative distribution associated with the upgrading of warning devices from flashing lights to gates. For instance, the 5th percentile value of CMF based on the cumulative normal distribution is 0.588, representing about a 41% reduction in collisions. The same percentile value for the cumulative beta is 0.521, representing about a 48% reduction in collisions. The 95th percentile values for the normal and beta cumulative distributions are estimated to be 0.751 (i.e. a 25% collision reduction) and 0.711 (i.e. a 29% collision reduction), for the normal and beta cumulative distributions, respectively. If we determine the CMF estimates based on the normal rather than the beta to represent the effectiveness of the upgrade from flashing lights to gates, a more conservative (lower safety benefit) result would be obtained.

Some countermeasures, such as Photo/Video enforcement, have not been introduced in Canadian inventory data, as a result, we could not make any inference from data likelihood. In this case, instead of employing Bayesian data fusion we recommend relying on the priors to represent full countermeasure effects. This would be appropriate until we obtain additional data likelihood inferences for this countermeasure. In a similar vein, if we do not have a priori knowledge about a given countermeasure (e.g. changes in train operating speeds) but we have current knowledge from the data, we can produce estimates of countermeasure effects based on the data likelihood estimates alone.

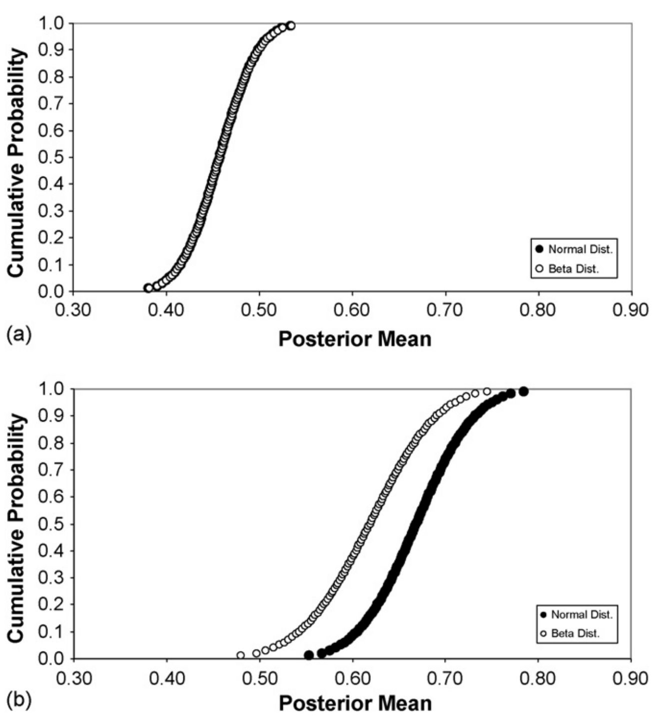

Fig. 1. Cumulative density functions based on the two different distributions: (a) CMF for the elimination of whistle prohibition and (b) CMF for the upgrading from flashing lights to gates.

5. Conclusions

Bayesian data fusion requires two important sources of information to obtain statistical estimates of countermeasure effects, a priori and data likelihood inputs. The approach suggested in this paper has a number of practical advantages for guiding decisions as to the merits of different countermeasures applied to specific crossings:

(1) It integrates results from previous studies of countermeasure effects with direct analysis of Canadian crossings collision experience, using a formal Bayesian data fusion procedure.
(2) It incorporates previous CMF findings from different model sources by systematically weighting the estimates on the basis of published model reliability. The use of weights contributes to more sound prior estimate of countermeasure effects, based on engineering experience.
(3) It allows for the adoption of different probability distributions for representing priors and data likelihoods. The use of normal and beta distribution in this study yielded modest differences in CMF for the upgrade in warning devices at the example crossing. To generalize this finding more research is required.
(4) Inasmuch as we used three different collision prediction models to obtain data likelihood estimates for the Canadian data, the proposed method yields countermeasure effects that are more reflective of a larger array of factors than is possible from a single model. This reduces problems of mis-specification commonly associated with these types of models.
(5) The proposed model provides tailored information about the effect of countermeasures for specific crossings of interest. This method employs data likelihood as an input, based on a series of collision prediction models developed for the Canadian collision database.
(6) The model formally recognizes uncertainty in the estimated countermeasure effects. Output is reported in terms

of means, variance, and corresponding probability distributions.

In summary, the Bayesian data fusion method proposed in this paper has several advantages especially for evaluating countermeasure effects at different levels of aggregation from individual crossings to a given region. It provides a promising tool for engineers to make informed and objective safety decisions in the face of uncertainty.

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